Name: _____

Instructor: _____

Math 10560, Exam 1.
February 14, 2008

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.
- Trigonometric formulas are provided in the last page.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1. (a)   (b)   (c)   (d)   (e)

2. (a)   (b)   (c)   (d)   (e)

....

3. (a)   (b)   (c)   (d)   (e)

4. (a)   (b)   (c)   (d)   (e)

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5. (a)   (b)   (c)   (d)   (e)

6. (a)   (b)   (c)   (d)   (e)

....

7. (a)   (b)   (c)   (d)   (e)

8. (a)   (b)   (c)   (d)   (e)

....

9. (a)   (b)   (c)   (d)   (e)

Please do NOT write in this box.

Multiple Choice _____

10. _____

11. _____

12. _____

13. _____

Total _____

Multiple Choice

1.(6 pts) The function $f(x) = \ln x - \dfrac{1}{x}$ is one-to-one. Compute $(f^{-1})'(-1)$.

(a)   0

(b)   2

(c) $\dfrac{1}{2}$

(d)   1

(e)   does not exist

2.(6 pts) Differentiate the function
$$f(x) = (2x)^x .$$

(a) $f'(x) = (2x)^x \left(\ln(2x) + \dfrac{1}{2} \right)$

(b) $f'(x) = (2x)^x \cdot x \ln(2x)$

(c) $f'(x) = (2x)^x \left(\ln(2x) + 2 \right)$

(d) $f'(x) = (2x)^x \left(\ln(2x) + 1 \right)$

(e) $f'(x) = x(2x)^{x-1}$

3.(6 pts) Compute the integral
$$\int_0^{\ln 2} \frac{e^x}{1 + e^x} \, dx \ .$$

(a) $\ln 3$ (b) 1 (c) e^2 (d) 0 (e) $\ln\left(\frac{3}{2}\right)$

4.(6 pts) Simplify the expression
$$\log_2\left(\frac{2^{x^2+1}}{4^x}\right) \ .$$

(a) x^2 (b) $(x-1)^2$ (c) 2

(d) $x^2 - x + 1$ (e) 1

5.(6 pts) A savings account has a yearly interest rate of r. Let $y(t)$ be the balance of the savings account after t years, and suppose the compounding of interest on the account is such that $y(t)$ satisfies the condition $y'(t) = ry(t)$. For which value of r will your initial investment triple in 15 years?

(a) $r = \ln \dfrac{3}{15}$

(b) $r = 3^{1/5} - 1$

(c) cannot be determined

(d) $r = \ln 3 - 15$

(e) $r = \dfrac{1}{15} \ln 3$

6.(6 pts) Compute $\tan^{-1}\left(\tan \dfrac{7\pi}{5}\right)$.

(a) $-\dfrac{7\pi}{5}$

(b) $-\dfrac{2\pi}{5}$

(c) $\dfrac{2\pi}{5}$

(d) does not exist

(e) $\dfrac{7\pi}{5}$

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7.(6 pts) Simplify $\sec(\tan^{-1} x)$.

(a) $\dfrac{x}{\sqrt{1+x^2}}$
  (b) $\dfrac{\sqrt{1+x^2}}{x}$
  (c) $\sqrt{1+x^2}$

(d) $\dfrac{1}{x}$
  (e) $\dfrac{1}{\sqrt{1+x^2}}$

8.(6 pts) Find the limit
$$\lim_{x\to 0} \frac{\sinh(x) - x}{x^3} \ .$$

(a) $\dfrac{1}{3}$
  (b) $\dfrac{1}{6}$
  (c) 0
  (d) $-\dfrac{1}{6}$
  (e) ∞

9.(6 pts) Evaluate the integral

$$\int_0^{\pi/2} \sin^3(x)\cos^3(x)dx \ .$$

(a) $\dfrac{1}{12}$

(b) $\dfrac{\pi^4}{64}\left(1 - \dfrac{\pi^2}{6}\right)$

(c) $\dfrac{5}{12}$

(d) $-\dfrac{5}{12}$

(e) $-\dfrac{1}{12}$

Partial Credit
You must show your work on the partial credit problems to receive credit!

10. (12 pts.) Evaluate the limit
$$\lim_{x \to 0} \left(\cosh(x) \right)^{1/x^2}.$$

11. (10 pts.) Compute the integral

$$\int_0^1 4\tan^{-1}(x)\,dx \ .$$

12. (12 pts.) Evaluate the integral

$$\int x(\ln x)^2 dx \ .$$

13. (12 pts.) Calculate the integral

$$\int \sqrt{4 - x^2} \, dx \ .$$

The following is the list of useful trigonometric formulas:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2\sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}\Big(\sin(x - y) + \sin(x + y)\Big)$$

$$\sin x \sin y = \frac{1}{2}\Big(\cos(x - y) - \cos(x + y)\Big)$$

$$\cos x \cos y = \frac{1}{2}\Big(\cos(x - y) + \cos(x + y)\Big)$$

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